

Structural Vulnerability Assessment of Electric Power Grids

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Abstract—Cascading failures are the typical reasons of black-outs in power grids. The grid topology plays an important role in determining the dynamics of cascading failures in power grids. Measures for vulnerability analysis are crucial to assure a higher level of robustness of power grids. Metrics from Complex Networks are widely used to investigate the grid vulnerability. Yet, these purely topological metrics fail to capture the real behaviour of power grids. This paper proposes a metric, the effective graph resistance, as a vulnerability measure to determine the critical components in a power grid. Differently than the existing purely topological measures, the effective graph resistance accounts for the electrical properties of power grids such as power flow allocation according to Kirchoff laws. To demonstrate the applicability of the effective graph resistance, a quantitative vulnerability assessment of the IEEE 118 buses power system is performed. The simulation results verify the effectiveness of the effective graph resistance to identify the critical transmission lines in a power grid.

I. INTRODUCTION

The electric power grid is crucial for economic prosperity, and national security. Disruptions to electrical power grids paralyse the daily life in modern societies causing huge economical and social costs for these societies [1]. The strong dependencies of other critical infrastructures such as telecommunications, transportation, and water supply on electric power grid amplifies the severity of large scale blackouts [2], [3]. The key importance of the electric power grid to society encourages further research into sustaining power system reliability and developing new methods to evaluate and mitigate the risk of cascading blackouts.

Power grid security is traditionally assessed relying on flow-based methods (e.g. $N-x$ contingency analysis [4]). The flow-based methods model the continuous and discrete dynamics of the components and solve non-linear algebraic equations to determine the electric power flow distribution over the grid components. A complete security assessment based on flow-based methods requires evaluating a combinatorial number of contingencies. However, this results in significant computational complexity and the associated computational time, enforcing power systems analysts to seek for alternatives [4].

The strong connection between the topology and the robustness of a grid prompts a vulnerability assessment from a topological perspective. The recent advances in the field of complex networks [5], [6] reveal its promising potential to

investigate power grids vulnerability at the systems level from a topological perspective.

Topological investigation of electrical power grids demonstrates that power grids belong to the classes of small world [6] networks, and they have scale free [5] characteristics, suggesting that power grids have "hub" components having significant criticality compared to the rest of the network. These components are crucial for the grid [7]. Their removal weakens the system robustness significantly, or results in the largest possible damage in network performance. Identifying these *critical components* in a power grid is a major concern for power system security and attracts significant attention from the power system research community [8], [9], [10]. Identifying these critical components in a power grid in advance enables power grid operators to improve system robustness by monitoring and protecting these components continuously.

To assess the structural vulnerability of power grids, several studies [9], [11], [12] have proposed extended topological approaches, while many others [13], [14], [15] have used graph theoretical metrics including average shortest path length [16], [17] and its derivatives such as efficiency [8]. These purely topological metrics consider only the structure of a power grid ignoring the electrical properties of the components in the network. However, in a power grid, the electric power is not governed only by the physical couplings (interconnections), but also by the electromagnetic couplings, and it flows through the network according to Kirchoff Laws. Therefore, the purely topological approaches considering only the topology fail to capture the real system behaviour. This paper proposes a metric, effective graph resistance, as a vulnerability measure to determine the critical lines in a power grid, while differently than the existing topological metrics, accounting for the power flow allocation according to Kirchoff Laws.

II. MODELLING CASCADING FAILURES IN POWER GRIDS

A power grid is a multi-layered network that is composed of three functional parts: generation, transmission, and distribution. Power is provided from generation buses to distribution stations through the transmission buses that are all interconnected via transmission lines. In a graph representation of a power grid, nodes represent generation, transmission, distribution buses, and substations, while links model the transmission lines and transformers. All the parallel transmission lines in the

system are represented by an equivalent single link in a graph representation. Additionally, the links in a graph representation are weighted by the admittance (or impedance) value of the corresponding transmission line.

The electrical properties of a grid including impedances, voltage levels at each individual power station, voltage phase differences between power stations and loads at terminal stations control the power flow in the grid. This paper estimates the flow values for each component in the network by using linear DC power flow equations [18] which is an approximation of nonlinear AC power flow equations [19]. In a DC model, the active power flow f_{ij} through a transmission line l_{ij} connecting node i and node j is related to the voltage phase values at both nodes i and j and the impedance of the line l_{ij} as follows:

$$f_{ij} = \frac{\theta_{ij}}{x_{ij}} = b_{ij}\theta_{ij} \quad (1)$$

where θ_{ij} is the voltage phase difference between node i and node j , x_{ij} is the reactance, and b_{ij} is the susceptance of l_{ij} . The entire system can be modelled solely by:

$$P_i = \sum_{j=1}^d f_{ij} = \sum_{j=1}^d b_{ij}\theta_{ij} \quad (2)$$

where P_i is the real power flow at node i and d is the degree of node i . In terms of matrices, Eq. (2) can be rewritten as:

$$\mathbf{P} = \mathbf{B}\boldsymbol{\theta} \quad (3)$$

where \mathbf{P} is the vector of real power injections, $\boldsymbol{\theta}$ contains the voltage phase values at each node, and \mathbf{B} is the bus susceptance matrix in which $B_{ij} = -\frac{1}{x_{ij}}$ and $B_{ii} = \sum_{j=1}^d -B_{ij}$. Since all the active power injections are known in advance, given the bus susceptance matrix \mathbf{B} , the voltage phase values at each node can be calculated directly by using:

$$\boldsymbol{\theta} = \mathbf{B}^{-1}\mathbf{P} \quad (4)$$

After obtaining the voltage angle values at each node, the power flow values through each line can be computed by using Eq. (1).

The maximum capacity of a line is defined as the maximum power flow that can be afforded by the line. The flow limit of a transmission line is imposed by thermal, stability or voltage drop constraints [20]. This paper assumes that the maximum capacity C_l of a line l is proportional to its initial load $L_{l,in}$ by a tolerance parameter α_l : $C_l = \alpha_l L_{l,in}$.

In a power grid, transmission lines are protected by relays and circuit breakers. A relay of a transmission line measures state variables (e.g. current), and compares them with a threshold value. When the threshold is violated, and this violation lasts long enough, the relay notifies a circuit breaker to trip the transmission line in order to prevent that the transmission line is permanently damaged due to e.g. overloading. This paper assumes a deterministic model for line tripping mechanism. A circuit breaker of a line l trips at the moment the load L_l of the line l exceeds its maximum capacity C_l : $|L_l/C_l| > 1$.

An initial outage of a component changes the balance of the power flow distribution over the grid and causes a redistribution of the power flow over the network. This dynamic response of the system to this triggering event might overload other parts in the network. The protection mechanism trips these newly overloaded components, and the power flow is again redistributed potentially resulting in new overloads. This cascade of failures continues until no more components are overloaded.

III. COMPLEX NETWORKS PRELIMINARIES

This section explains the relevant basic concepts from complex networks theory, introduces the effective graph resistance, and elaborates on how it is computed in electric power grids.

A. Complex networks basics

A network $G(N,L)$ consisting of a set of nodes N and links L , can fully be represented by its adjacency matrix. The *adjacency matrix* A of a simple, unweighted graph $G(N,L)$ is an $N \times N$ symmetric matrix reflecting the interconnection of the nodes in the graph. $a_{ij} = 0$ indicates that there is no edge between nodes i and j , otherwise $a_{ij} = 1$. In case of a weighted graph, the network is represented by the weighted adjacency matrix W where w_{ij} corresponds to the weight of the line between nodes i and j ; a weight can be a distance, cost, or delay.

The *Laplacian matrix* [21] Q is another way to fully characterize a graph, and defined as:

$$Q = \Delta - A \quad (5)$$

where Δ is the diagonal matrix of strengths of G : $\delta_i = \sum_j w_{ij}$. Hence, the Laplacian can be constructed as:

$$Q_{ij} = \begin{cases} \delta_i, & \text{if } i = j. \\ -w_{ij}, & \text{if } i \neq j \text{ and } (i, j) \in L \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

where δ_i is the strength of node i , and L is the set of links in G .

A *walk* between pair of nodes i and j is a set of nodes and links that begins at node i and ends at node j , while a *path* P_{ij} refers to a walk in which no nodes are revisited. The *path length* $l(P_{ij})$ is the sum of the weights of constituent edges in the path P_{ij} . The shortest path length $l(P_{ij}^*)$ is the minimizer of $l(P_{ij})$ over all P_{ij} . The *average shortest path length* l_G of a network G is defined as:

$$l_G = \frac{1}{N(N-1)} \sum_{i \neq j \in G} l(P_{ij}^*) \quad (7)$$

B. Effective graph resistance in power grids

Considering a network $G(L,N)$ with a Laplacian matrix constructed by the conductance values of the lines, the effective resistance [21] between a pair of nodes i and j R_{ij} is the potential difference between these nodes when a unit current is injected at node i and withdrawn at node j . The effective graph resistance is the sum of the individual effective

resistances between each pair of nodes in the network. The effective graph resistance can be computed in two different ways: (a) by aggregating the effective resistances between each pair of nodes, and (b) by the eigenvalues of the Laplacian matrix of the grid.

The required steps to compute the effective graph resistance based on pairwise effective resistances are (i) constructing the Laplacian matrix of the grid, (ii) determining the generalised inverse of the Laplacian matrix, (iii) computing effective resistances between each pair of nodes, and (iv) summing up the effective resistances.

The Laplacian matrix of a power grid Q reflects the interconnection of buses with transmission lines according to the description in Eq. 6. The weight w_{ij} corresponds to the susceptance [19] (i.e. inverse of reactance) value between nodes i and j . The Laplacian matrix constructed by the susceptance values is equivalent to the admittance matrix in the electrical power systems theory.

The *effective resistance* R_{ij} between nodes i and j is computed as:

$$R_{ij} = Q_{ii}^+ - 2Q_{ij}^+ + Q_{jj}^+ \quad (8)$$

where Q^+ is the generalized inverse of Q obtained by the Penrose pseudo-inverse operator [22].

Subsequently, the *effective graph resistance* R_G of a power network is computed by summing up all the effective resistances between all pairs in a network.

$$R_G = \sum_{i=1}^N \sum_{j=i+1}^N R_{ij} \quad (9)$$

Another way to compute the effective graph resistance of a power grid requires computation of the eigenvalues of the Laplacian matrix of the grid. This approach requires summing the reciprocal of the eigenvalues:

$$R_G = N \sum_{i=1}^{N-1} \frac{1}{\mu_i} \quad (10)$$

where μ_i is the i^{th} eigenvalue of the Laplacian matrix. This methodology is computationally more efficient, but it does not give any insight into the individual effective resistances between pairs of buses.

In a DC model of a electrical power grid, the effective resistance R_{ij} between buses i and j is equal to the equivalent impedance $Z_{eq,ij}$ between these buses. Fig. 1 illustrates the case.

IV. EFFECTIVE GRAPH RESISTANCE AS A METRIC TO ASSESS GRID VULNERABILITY

In purely topological approaches such as shortest path length, electrical power is assumed to follow the shortest or the most efficient path. Relying on this assumption, a shortest path between a pair of nodes is determined. The criticality of a transmission line is then quantified based on the impact of its removal on the network average shortest path length. However, electric power is not governed only by topological, but also

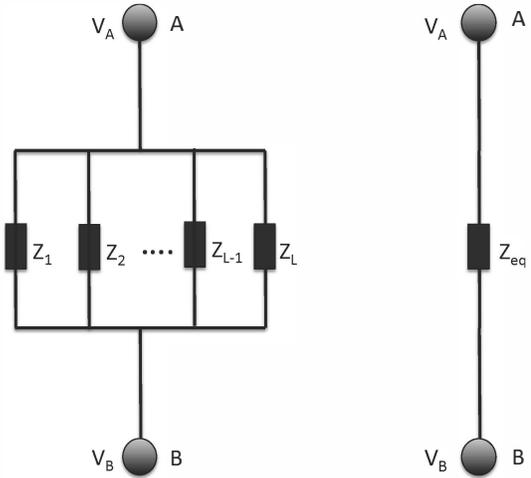


Fig. 1. Multiple L branches between nodes A and B with impedances Z_1, Z_2, \dots, Z_L can be replaced by one single branch with an impedance value $Z_{eq,AB}$. $Z_{eq,AB}$ is related to the potential difference between A and B as: $V_A - V_B = IZ_{eq,AB}$. Assuming a unit electric current I , the equivalent impedance equals the potential difference between nodes A and B , and also, per definition, the effective resistance between nodes A and B .

by the electrical properties of the grid. The electric power flows through all the possible paths rather than following one single path (e.g. the shortest path). Consequently, these purely topological approaches, that consider only the topology of a grid and ignore the electric characteristics, fail to capture the real behaviour of the grid in terms of robustness and vulnerability.

In power grids, utilization of multiple paths precludes the existence of a physical shortest path between two buses. However, the concept of *effective resistance* makes it possible to determine a distinct *electrical path* between two nodes by conceptually replacing the multiple paths between two nodes with a single equivalent path. The effective resistance between two nodes is the total cost incurred to transfer electric power between these nodes. Consequently, the effective resistance is the (real) electrical path length between two nodes, and can replace its purely topological counterpart (the shortest path length) for a realistic vulnerability analysis of power grids.

The concept of effective resistance makes it possible to construct the *electrical topology* of a power grid. An electrical topology of a power grid shows the electrical connections between buses, rather than the physical connections as a physical topology does. In an electrical topology of a power grid, the nodes represent generation, transmission, distribution buses, and substations while a link between nodes i and j corresponds to the effective resistance R_{ij} . Fig. 2 shows the physical and the electrical topology of the IEEE 14 power system [23].

In a physical power grid topology, the existence of parallel paths between two nodes, and a homogeneous distribution of their impedance values result in a smaller effective resistance between these two nodes. The number of parallel paths in the physical topology refers to the number of redundant

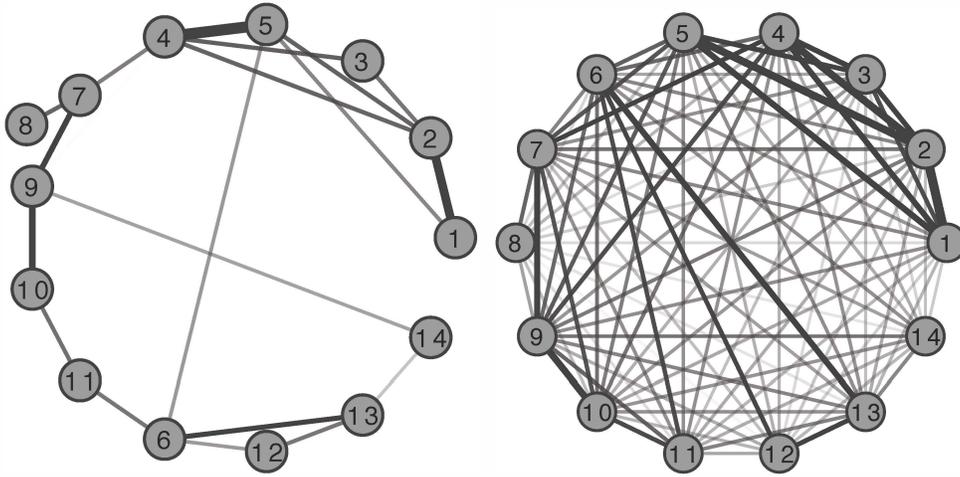


Fig. 2. The physical (left) and the electrical (right) topology of the IEEE 14 power system. In the physical topology the conductances, and in the electrical topology the effective conductances (i.e. $1/R_{ij}$), are used as weights for a better illustration. In the physical topology, a relatively thicker and more visible line corresponds to a stronger connection (i.e. a smaller effective resistance), while a relatively thinner and less visible line corresponds to a relatively weaker connection (i.e. larger effective resistance).

(backup) paths. In case of a failure in one of the paths between two nodes, the power flow carried by the rendered path is distributed over the backup paths. Therefore, a higher number of backup paths implies a more robust network against cascading failures due to line overloads. On the other hand, a relatively more homogeneous distribution of the impedance values results in a relatively more homogeneous distribution of power flow over these parallel paths increasing the robustness of the power grid against cascading failures [24], [25]. Therefore, a power grid with a relatively smaller effective graph resistance implies a relatively more robust power grid with respect to cascading failures. Koç et al. [26] verifies the relationship between R_G and the robustness of a power grid.

From an electrical topology point of view, a relatively small R_G results in relatively strong local electrical connections (e.g. electrical connection between buses 1 and 2 in the electrical topology in Fig. 2) and relatively strong remote electrical connections (e.g. electrical connection between buses 6 and 13 in the electrical topology in Fig. 2) between buses. The strong local electrical connections in a part of a power grid allows a better accommodation of power flow in that area, increasing the local robustness [24], [25]. On the other hand, the strong remote electrical connections between distant buses enable transfer of electrical power flow from a region of the grid to another region resulting in a better accommodation of power flow over the entire power grid. In case of a congestion in one part of the grid (e.g. as a result of a failure), the excess power in the associated part is easily transferred to the rest of the grid, enabling the utilization of the redundant capacity in the rest of the grid. Accordingly, relatively stronger electrical connections between buses (i.e. a relatively smaller effective graph resistance) allow a better accommodation of power flow and increase the ability of a power grid to distribute the excess power over the rest of the network ensuring a higher tolerance

against local failures.

The effective graph resistance locates the transmission lines that are contributing most to the electrical connections in a power grid. Removal of these lines reduces the ability of a power grid to better accommodate power flow, hence, the attack tolerance of the grid. Accordingly, these lines are the lines with the highest criticality in a power grid and a robust operation of a power grid requires continuous monitoring and protection of these lines.

V. USE CASE: IDENTIFYING THE CRITICAL LINES IN A POWER GRID

This section demonstrates how the effective graph resistance is used as a metric to assess power grid vulnerability, and to determine the critical components for the system.

For a quantitative criticality analysis, the IEEE 118 buses power system [23] is considered. To assess the criticality of a transmission line based on R_G , this section deploys an analogous approach to the one given in [7], [27]: the criticality of a transmission line l in a grid G is determined by the relative increase in the effective graph resistance ΔR_G^l that is caused by the deactivation of line l :

$$\Delta R_G^l = \frac{R_{G-l} - R_G}{R_G} \quad (11)$$

where R_{G-l} is the effective graph resistance of the grid that is obtained from G by removing l .

The original grid R_G , and the impact of each individual line removal on R_G are computed. By substituting these values in Eq. 11, the impact of each transmission line on the grid robustness (ΔR_G^l) is assessed. At the same time, the criticality of each transmission line is also quantified based on l_G (Δl_G^l) by following the same approach. Fig. 3 shows the results for all transmission lines in the IEEE 118 power system, while

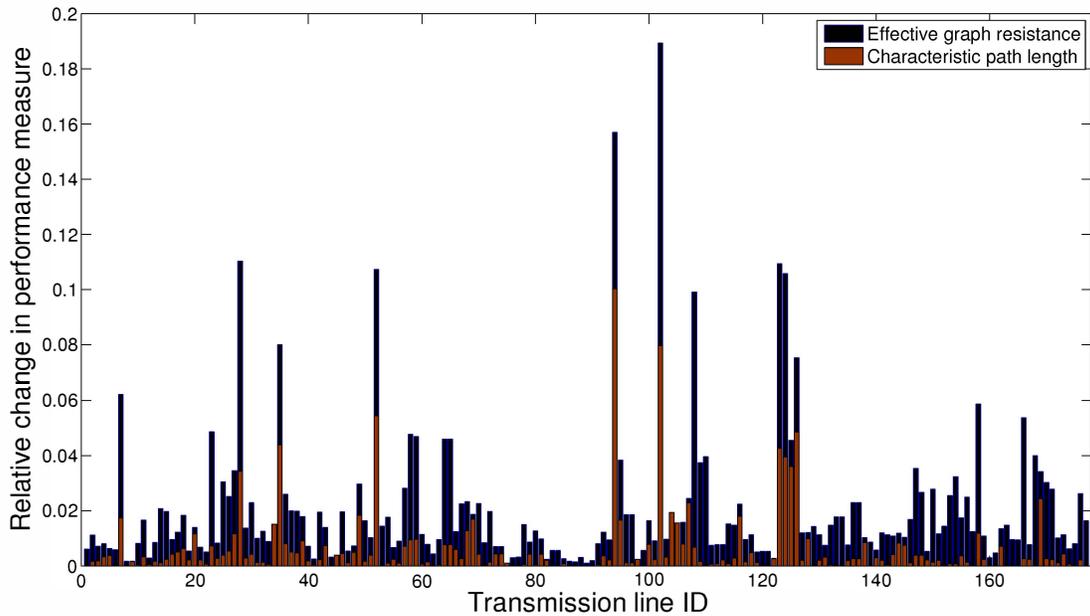


Fig. 3. Relative increase in effective graph resistance and in average shortest path for IEEE 118 test case

TABLE I
MOST CRITICAL 10 TRANSMISSION LINES IN IEEE 118 POWER SYSTEM
BASED ON R_G AND l_G

Line ID	$\Delta R_G^l(\%)$	Line ID	$\Delta l_G^l(\%)$
l_{65-68}	18.94	l_{38-65}	10.02
l_{38-65}	15.70	l_{65-68}	7.97
l_{23-24}	11.03	l_{30-38}	5.44
l_{68-81}	10.94	l_{82-83}	4.86
l_{30-38}	10.70	l_{8-30}	4.39
l_{80-81}	10.57	l_{68-81}	4.27
l_{70-71}	9.90	l_{80-81}	3.95
l_{8-30}	8.30	l_{77-82}	3.61
l_{82-83}	7.52	l_{23-24}	3.44
l_{8-5}	6.22	$l_{103-110}$	2.43

Table I shows the 10 most critical lines identified by R_G and l_G .

In Fig. 3 and Table I, the line criticality analysis based on R_G shows that the transmission line with the ID of 104 (connecting bus 65 to bus 68) is the most critical line for the IEEE 118 power system. Deactivation of line 104 causes nearly a 19% increase of the effective graph resistance of the grid. On the other hand, the analysis based on the average shortest path length suggests that the transmission line with the ID of 96 (connecting bus 38 to bus 65) is the most critical line and its removal results in nearly 10% increase of average shortest path length of the grid topology.

The top 10 most critical lines according to R_G are almost the same as the top 10 lines based on l_G . However, these lines have different rankings in their criticality. The purely topological l_G ignores the electrical properties and identifies the criticality of transmission lines purely based on the centrality of their location in the grid. On the other hand, R_G incorporates the impact of the electrical properties

of transmission lines in addition to the importance of their central locations. This makes the difference between the two approaches and results in different ranking of importances of these components.

To validate the results from Table I, the IEEE 118 power system is attacked by removing the critical lines identified by R_G and l_G , and the damage is quantified. The simulations are performed by MATCASC [28], a MATLAB based cascading failures analysis tool implementing the model in Sec. II. The damage caused by the cascade is quantified in terms of normalized served power demand: served power demand divided by the total power demand in the grid. The attack vulnerability of the grid differs under various operative states for the same attack. To capture the vulnerability of the system under different operative states, the normalized served power demand is averaged over 100 random instances of power demand, varying in the interval of $[P_d, 3P_d]$ (P_d is the base power demand given in [23]).

Initially, the top 10 critical lines identified by R_G (see Table I) are attacked one after the other. After each successive attack, the served power demand in the grid is quantified. The same analysis is performed by attacking the top 10 critical lines based on l_G (see Table I), and 10 lines based on random removals. Fig. 4 shows how the fraction of served power demand decreases after each successive attack. The largest damage results after an attack strategy based on R_G validating the effectiveness of R_G to identify the critical transmission lines in a power grid.

VI. CONCLUSION AND DISCUSSION

This paper proposes the *effective graph resistance* R_G as a global vulnerability measure for power grids. Based on R_G , the critical transmission lines in a power grid are determined.

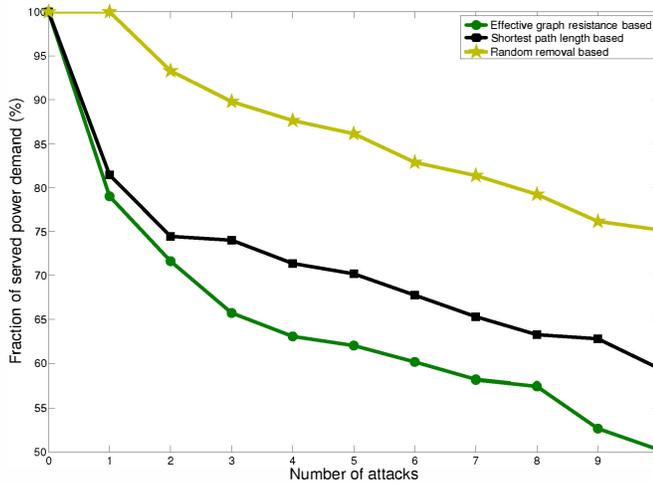


Fig. 4. Effectiveness of attacks based on effective graph resistance, average shortest path length, and random removals for IEEE 118 power system

The proposed metric R_G serves as a better tool to assess the grid vulnerability compared to traditional average shortest path length by incorporating the electrical properties of power grids such as power flow allocation according to Kirchoff Laws. The proposed approach is applied on the IEEE 118 power system to determine the critical components. Results are compared to the traditional topological metric average shortest path length. Simulation results verify the effectiveness of R_G as a measure to assess power grid vulnerability.

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